DIRECT GENERATION OF FINITE ELEMENT MESHES OF COMPOSITE MICRO AND MESOSTRUCTURE FROM 3D IMAGING: APPLICATION TO FLOW COMPUTATION

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Keywords: *Finite elements, mesh generation, composite microstructure, X-Ray tomography.*

Introduction

In recent years, imaging techniques, like X-ray tomography, have been used to obtain precise geometrical and topological information. For scientific computation, accuracy of numerical representations may play a key role: image acquisition and its efficient transposition on a numerical sample have shown to be of prime importance. The passage from the image to a finite element mesh allows on one hand, the construction of a numerical representation but, on the other hand, may also reduce the size of the stored data. The first step (before building a mesh) is usually to perform the image segmentation, that is, to partition the image into its multiple objects, so that it becomes easier to analyse it [1]. The result of this segmentation can be regions of the image where the pixels are similar with respect to a characteristic or property (like, for example, the colour), or extracted contours. Secondly, a mesh is built from the segmented image until the optimal mesh is obtained, by using different techniques. One class of segmentation techniques is based on an implicit description of the phases, through level set functions [2-3]. In this paper, we propose to simultaneously segment the image and produce an adapted mesh adequate to perform numerical simulations using the image data, based on the reinitialization of an implicit function coupled to an anisotropic mesh adaptation technique [4]. Simulations performed concern mainly multiphase flow and, in particular, flows through solid fibrous media.

The immersed image method and anisotropic mesh adaptation

Let us begin by introducing the Immersed Image Method, which is the interpolation of the image's pixel/voxel values (a sort of topographic distance function), designated $\tilde{u}$, on the nodes of an initial mesh, providing the distributed field, u. The obtained field can approximately represent the original image, depending on the discretization and on the overall number of nodes. Optimization of the number of necessary nodes can be performed through anisotropic mesh adaptation using an appropriate error estimator on u [4], so that the obtained field accurately represents the original image (but is also enough enriched to perform the necessary computation). Figure 1 illustrates our purpose.

Segmentation through reinitialization coupled to adaptation

To further progress towards segmentation, we choose to adapt the mesh not on the interpolated topographic distance function (the pixel/voxel value), but on its regularized Heaviside, u_ε. To build this Heaviside, which is a smooth hyperbolic tangent varying on a certain thickness ε around a predetermined value of the topographic distance, we have implemented a reinitialization method, by solving an Hamilton-Jacobi type equation, ensuring that its gradient is preserved, such that $\|\nabla u_\varepsilon\| = 1 - \left(\frac{u_\varepsilon}{\varepsilon}\right)^2$.

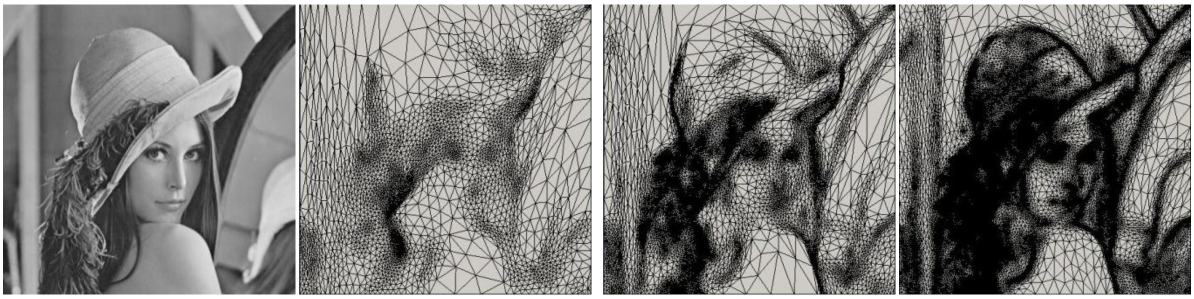

Figure 1: *Initial image and obtained anisotropic mesh, for four different numbers of imposed nodes*

This procedure, coupled to mesh adaptation, allows segmentation but allows also to build up a mesh with a smooth representation of a phase function distribution, usable in the aimed type of application (Figure 2, in the middle).

Numerical results on composite flow computation

Flow is then computed in an Eulerian framework, by solving the Navier-Stokes equations with heterogeneous material properties, corresponding to each phase. An image (900x900x220 voxels) issuing from 3D X-Ray tomography [5]) concerning a mat-reinforced sample is treated and u_ε built, a phase function equal to ε inside the fibres (solid) and -ε in the resin (fluid) phases. Finite element resolution of these equations is done using a mixed stabilized finite element method, using 96 CPUs and a generated mesh of 5 millions of nodes. Results detailed in Figure 2 were obtained by imposing a pressure gradient through the sample on one direction and exhibit the type of computation that can be performed.

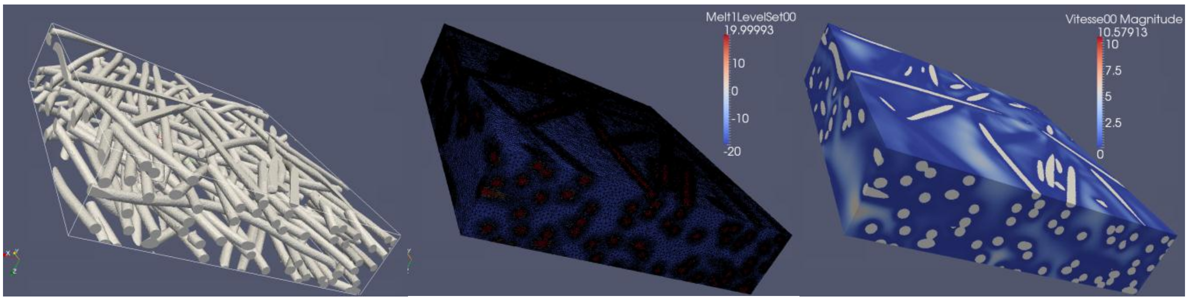

Figure 2: *Flow on a composite image: iso-value 0 of the phase function* u_ε *, mesh and computed velocity-norm*

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